Theory of turbulence augmentation across hypersonic shock waves

74th Annual Meeting of the APS Division of Fluid Dynamics

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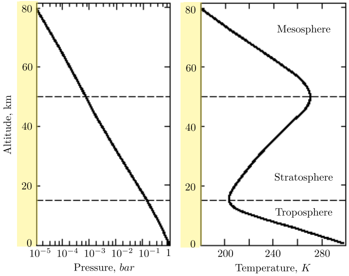
Phoenix | November 21-23, 2021

Huete: Spanish MCINN and BBVA Foundation (Leonardo Grant)
Urzay: US AFOSR and US DoE/NNSA

Motivation



$$\frac{\Delta p}{p} \sim \frac{\Delta \rho}{\rho} = O(10^5), \qquad \frac{\Delta T}{T} = O(1).$$

In hypersonic flight near the ground, the Reynolds number becomes large because of the comparatively larger densities

$$\frac{\Delta Re}{Re} = O(10^5).$$

Urzay, J., & Di Renzo, M. (2021). Annual Research Briefs, Center for Turbulence Research, 7-32.

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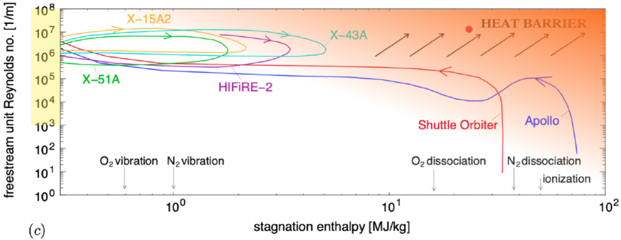

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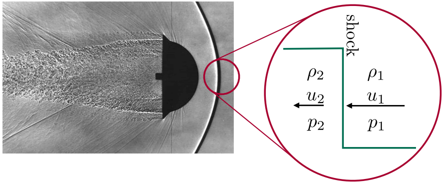
Hypersonic flight at low altitudes is characterized by:

- High free-stream Mach numbers $Ma \geq 5$
- High free-stream and post-shock unit Reynolds numbers $Re \sim 10^7 - 10^9$ m^{-1}
- High stagnation enthalpies $h_0 \sim 5 - 30$ MJ/kg
- Small mean free paths $\lambda \sim 0.1\ \mu$m

- large normal Mach numbers
- turbulent boundary layers
- much higher than the vibrational specific energies of O_2 and N_2
- short vibrational relaxation distances

Effects of dissociation and vibrational excitation on the mean post-shock quantities



Integral conservation equations accross shock waves in dissociating gases

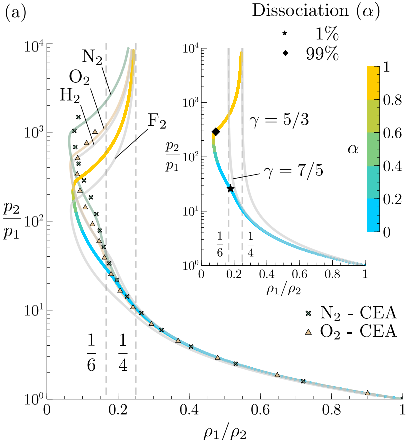
$$\rho_1 u_1 = \rho_2 u_2,$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$

$$e_1 + p_1/\rho_1 + u_1^2/2 = e_2 + p_2/\rho_2 + u_2^2/2 + q_d,$$

Upstream flow is sufficiently cold to be approximated as

$$e_1 = (5/2) R_{g,\mathrm{A}_2} T_1, \quad p_1 = \rho_1 R_{g,\mathrm{A}_2} T_1.$$



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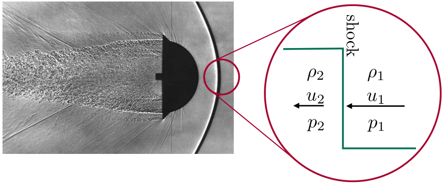

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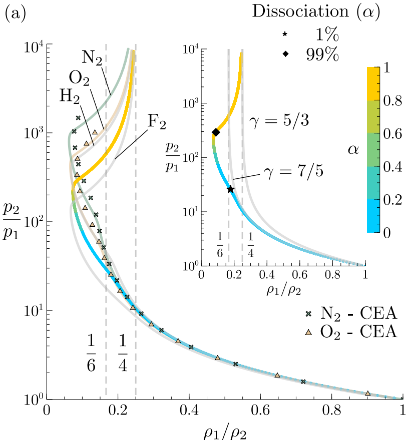
$$p_2 = \rho_2 R_{g,\mathrm{A}_2} T_2 (1+\alpha), \quad q_d = \alpha R_{g,\mathrm{A}_2} \Theta_d,$$

$$e_2 = R_{g,\mathrm{A}_2} T_2 \left[3\alpha + (1-\alpha)\left(\frac{5}{2} + \frac{\Theta_v/T_2}{\mathrm{e}^{\Theta_v/T_2} - 1} \right) \right],$$

$$\frac{\alpha^2}{1-\alpha} = G m \Theta_r \left(\frac{\pi m k_B}{\hbar^2} \right)^{3/2} \frac{\sqrt{T_2}}{\rho_2} \mathrm{e}^{-\frac{\Theta_d}{T_2}} \left(1 - \mathrm{e}^{-\frac{\Theta_v}{T_2}} \right),$$



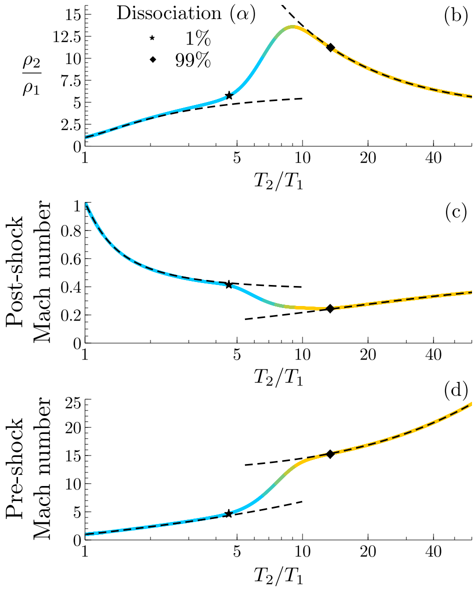
where α is the degree of dissociation, defined as the mass fraction of A atoms in the reaction $\mathrm{A}_2 \rightleftharpoons \mathrm{A} + \mathrm{A}$, that must be solved with the aid of the chemical equilibrium condition.

Effects of dissociation and vibrational excitation on the mean post-shock quantities

Endothermicity due to dissociation and vibrational excitation does the following:

- increases the mean post-shock density
- decreases the mean post-shock velocity
- decreases the mean post-shock Mach number
- decreases the mean post-shock temperature

Jump conditions



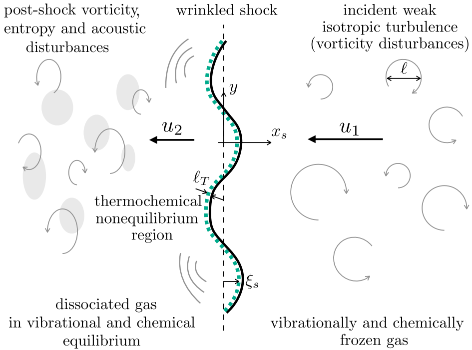
LIA of turbulence interacting with hypersonic shocks

Considering:

- turbulence is comprised of small fluctuations,
- Kovasznay decomposition into vortical, entropic and acoustic modes,

we can solve this problem analytically by using

- linearized Rankine-Hugoniot relations,
- linearized Euler equations in the post-shock gas.



Limits of validity

Assumptions standard LIA:

(a) $\mathrm{rms}(u_\ell) \ll a_1$ and a_2,

(b) $\xi_s \ll \ell$,

(c) $\ell/u_\ell \ll \ell^2/\nu$.

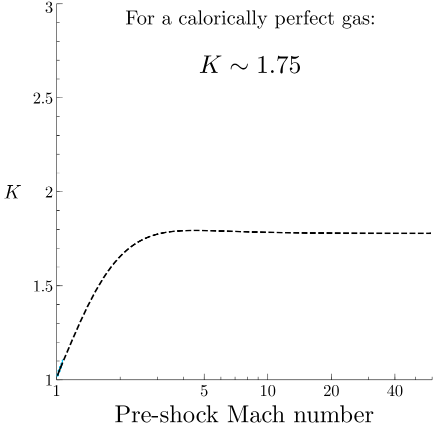
With thermochemical effects:

(d) $\ell_T \ll \ell$

For a given ℓ, this condition becomes increasingly more accurate as the altitude decreases.

LIA of turbulence interacting with hypersonic shocks: Calorically Perfect Gas

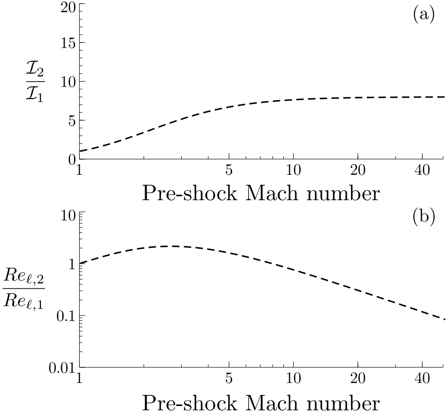
Turbulent Kinetic Energy



For a calorically perfect gas:

$$K \sim 1.75$$

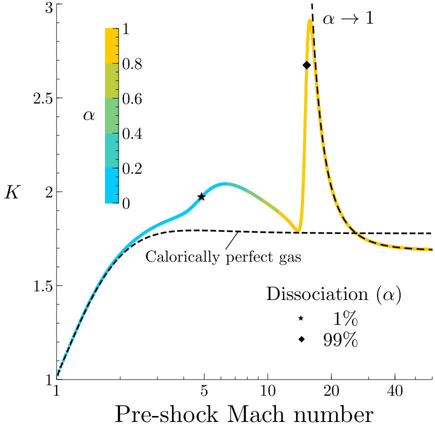
Turbulence intensity and Turbulent Reynolds number



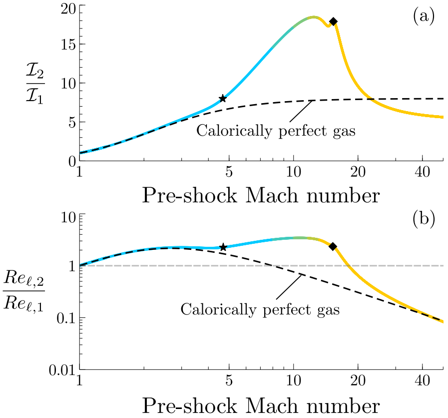
At hypervelocities ($Ma \gtrsim 10$), the calorically perfect gas approximation predicts a saturation in the amplification of kinetic energy and turbulence intensity, along with a decrease in the turbulent Reynolds number accross the shock.

LIA of turbulence interacting with hypersonic shocks: Vibrationally Excited, Dissociating Gas

Turbulent Kinetic Energy



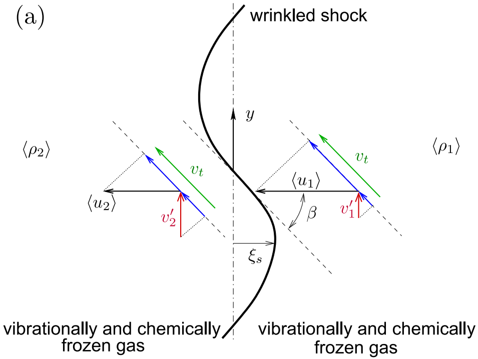
Turbulence intensity and Turbulent Reynolds number



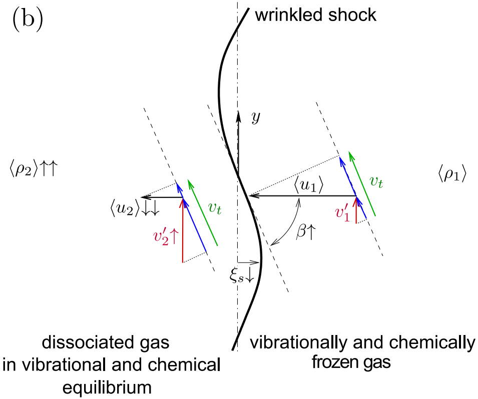
In contrast, the incorporation of dissociation and vibrational excitation predicts larger kinetic energy and turbulence intensity amplification rates, along with an increase in the turbulent Reynolds number accross the shock.

LIA mechanism of turbulence amplification across hypersonic shocks

Calorically perfect gas



Vibrationally Excited, Dissociating Gas



Conservation of tangential momentum dictates that the transverse velocity fluctuations should increase across the shock – these are larger at hypersonic velocities because of the associated larger post-shock densities induced by endothermic thermochemical effects.

Conclusions

Key takeaways

- Significant departures from calorically perfect gas behaviour can be observed in the solution even at modest degrees of dissociation of 1%.
- A turning point in the Hugoniot curve is observed at approximately Mach 13 and 70% degree of dissociation.
- The amplification of TKE doubles that observed in calorically perfect gases, with most of the content of TKE downstream in form of vortical modes.
- The turbulent Reynolds number is amplified across the shock at hypersonic Mach numbers in the presence of dissociation and vibrational excitation, as opposed to the attenuation observed in the calorically perfect case.

- **Thermochemical effects arising at hypersonic velocities appear to enhance turbulent fluctuations in the post-shock gas.**

Huete, C. et al. (2021). Thermochemical effects on hypersonic shock waves interacting with weak turbulence. Physics of Fluids, 33(8), 086111